

A **solution of an equation** is the value (or values) of the variable that makes the equation true. To find a solution, you can use properties of equality to form equivalent equations. **Equivalent equations** are equations that have the same solution (or solutions).

Addition Property of Equality

For all real numbers a , b , and c ,
if $a = b$, then $a + c = b + c$.

Example $8 = 5 + 3$, so
 $8 + 4 = 5 + 3 + 4$.

Multiplication Property of Equality

For all real numbers a , b , and c ,
if $a = b$, then $a \cdot c = b \cdot c$.

Example $\frac{6}{2} = 3$, so $\frac{6}{2} \cdot 2 = 3 \cdot 2$.

Subtraction Property of Equality

For all real numbers a , b , and c ,
if $a = b$ then $a - c = b - c$.

Example $8 = 5 + 3$, so
 $8 - 2 = 5 + 3 - 2$.

Division Property of Equality

For all real numbers a , b , and c ,
with $c \neq 0$, if $a = b$ then $\frac{a}{c} = \frac{b}{c}$.

Example $3 + 1 = 4$, so $\frac{3+1}{2} = \frac{4}{2}$.

One way to find the solution of an equation is to get the variable alone on one side of the equal sign. You can do this using **inverse operations**, which are operations that undo one another. Addition and subtraction are inverse operations. Multiplication and division are also inverse operations.

1 EXAMPLE Solving Using Addition or Subtraction

a. Solve $x - 3 = -8$.

$$x - 3 + 3 = -8 + 3 \quad \text{Add 3 to each side of the equation.}$$

$$x = -5 \quad \text{Simplify.}$$

b. Solve $g + 7 = 11$.

$$g + 7 - 7 = 11 - 7 \quad \text{Subtract 7 from each side of the equation.}$$

$$g = 4 \quad \text{Simplify.}$$

2 EXAMPLE Solving Using Multiplication or Division

a. Solve $\frac{3}{4}x = 9$.

$$\frac{4}{3}\left(\frac{3}{4}x\right) = \frac{4}{3}(9) \quad \text{Multiply each side by } \frac{4}{3}, \text{ the reciprocal of } \frac{3}{4}.$$

$$x = 12 \quad \text{Simplify.}$$

b. Solve $-96 = 4c$.

$$\frac{-96}{4} = \frac{4c}{4} \quad \text{Divide each side by 4.}$$

$$-24 = c \quad \text{Simplify.}$$

Solve each equation.

1. $x - 8 = 0$

2. $c - 4 = 9$

3. $-4 = \frac{2}{5}a$

4. $-8n = -64$

5. $b + 5 = -13$

6. $6 = x + 2$

7. $-7y = 28$

8. $-101 = -\frac{r}{3}$

9. $67 = w - 65$

10. $5b = 145$

11. $\frac{m}{7} = 12$

12. $-4 = k + 19$

EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice*.

Practice and Problem Solving

A Practice by Example

Example 1
(page 120)



Solve each equation. Check your answer.

1. $1 + \frac{a}{5} = -1$
2. $2n - 5 = 7$
3. $-1 = 3 + 4x$
4. $\frac{y}{2} + 5 = -12$
5. $3b + 7 = -2$
6. $\frac{x}{3} - 9 = 0$
7. $14 + \frac{h}{5} = 2$
8. $-10 = -6 + 2c$
9. $\frac{m}{8} + 4 = 16$
10. $\frac{a}{4} - 21 = 7$
11. $3x - 1 = 8$
12. $10 = 2n + 1$
13. $35 = 3 + 5x$
14. $41 = \frac{2}{5}x - 7$
15. $-3 + \frac{m}{3} = 12$
16. $9 + \frac{n}{5} = 19$
17. $-x - 4 = -20$
18. $-y + 10 = 25$
19. $5 = -z - 3$
20. $9 = -x + 8$

Example 2
(page 120)

Define a variable and write an equation for each situation. Then solve.

21. **Donations** A library receives a large cash donation and uses the funds to double the number of books it owns. Then a book collector gives the library 4028 books. After this, the library has 51,514 books. How many books did the library have before the cash donation and the gift of books?
22. **Cooking** Suppose you are helping to prepare a large meal. You can peel 2 carrots per minute. You need 60 peeled carrots. How long will it take you to finish if you have already peeled 18 carrots?
23. **Cell Phones** One cell phone plan costs \$39.95 per month. The first 500 minutes of usage are free. Each minute thereafter costs \$.35. Write a rule that describes the total monthly cost as a function of the number of minutes of usage (over 500 minutes). Then find the number of minutes of usage over 500 minutes for a bill of \$69.70.

Example 3
(page 121)

Justify each step.

24. $\frac{x}{5} + 9 = 11$
 $\frac{x}{5} + 9 - 9 = 11 - 9$
 $\frac{x}{5} = 2$
 $5\left(\frac{x}{5}\right) = 5(2)$
 $x = 10$
25. $-y - 5 = 11$
 $-y - 5 + 5 = 11 + 5$
 $-y = 16$
 $-1(-y) = -1(16)$
 $y = -16$
26. $18 - n = 21$
 $18 - n - 18 = 21 - 18$
 $-n = 3$
 $-1(-n) = -1(3)$
 $n = -3$
27. $12 - 2h = 8$
 $12 - 2h - 12 = 8 - 12$
 $-2h = -4$
 $\frac{-2h}{-2} = \frac{-4}{-2}$
 $h = 2$

Example 4
(page 121)

B Apply Your Skills

Solve each equation.

28. $\frac{5}{7}x + \frac{1}{7} = 3$
29. $\frac{a}{5} + 15 = 30$
30. $-\frac{1}{5}t - 2 = 4$
31. $-6 + 6z = 0$
32. $3.5 + 10m = 7.32$
33. $7 = -2x + 7$
34. $\frac{1}{2} = \frac{2}{5}c - 3$
35. $10.7 = -d + 4.3$
36. $0.4x + 9.2 = 10$
37. $4x + 92 = 100$
38. $-t - 0.4 = -3$
39. $-10t - 4 = -30$

Solve each equation. Justify each step.

40. $8 + \frac{c}{-4} = -6$
41. $7 - 3k = -14$
42. $14 = 6 - 2p$
43. $\frac{-y}{2} + 14 = -1$

44. **Multiple Choice** Beneath Earth's surface, the temperature increases 10°C every kilometer. Suppose that the surface temperature is 22°C , and the temperature at the bottom of a coal mine is 45°C . Which equation could be used to find the depth d of the coal mine?

(A) $10d + 22 = 45$

(B) $45d - 10 = 22$

(C) $d = 22 + 10$

(D) $22 = 10d + 45$

45. **Insurance** One health insurance policy pays people for claims by multiplying the claim amount by 0.8 and then subtracting \$500. Write a rule that describes the insurance payment as a function of the claim amount. Then find the claim amount for an insurance payment of \$4650.

46. **Library** The Library of Congress in Washington, D.C., is the largest library in the world. It contains nearly 128 million items. The library adds about 10,000 items to its collection daily. Write a rule that describes the number of items in the Library of Congress as a function of the number of items added daily. Then find the number of days it will take the library to reach about 150 million items.



Real-World Connection

Since 1950, the size of the collections and the size of the staff of the Library of Congress have tripled.

Solve each equation. (*Hint: As your first step, multiply each side by the denominator of the fraction.*)

47. $\frac{x+2}{9} = 5$

48. $\frac{y+1}{3} = 2$

49. $\frac{a-10}{-4} = 2$

50. $\frac{b-7}{2} = 6$

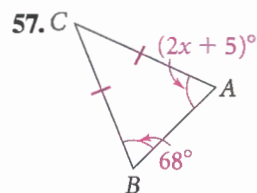
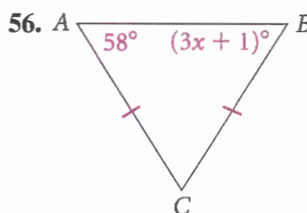
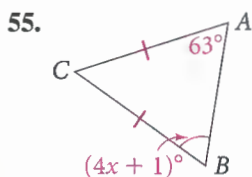
51. $\frac{x-5}{2} = 10$

52. $\frac{x-3}{7} = 12$

53. $\frac{x+4}{3} = -8$

54. $\frac{x+6}{4} = -7$

55. **Geometry** In each triangle, the measure of $\angle A =$ the measure of $\angle B$. Find the value of x .



58. **Writing** Miles has saved \$40. He wants to buy a CD player for \$129 in about four months. To find how much he should save each week, he wrote $40 + 16x = 129$. Explain his equation.

Error Analysis What is the error in the work? Solve each equation correctly.

59.
$$\begin{aligned} 12 - 3y &= 15 \\ 3y &= 3 \\ y &= 1 \end{aligned}$$

60.
$$\begin{aligned} \frac{m}{3} - 9 &= -21 \\ \frac{m}{3} - 9 + 9 &= -21 + 9 \\ \frac{m}{3} &= -12 \\ m &= -4 \end{aligned}$$

61. **Open-Ended** Write a problem that you can model with a two-step equation. Write an equation and solve the problem.

62. You can find the value of each variable in the matrices below by writing and solving equations. For example, to find the value of a , you solve the equation $2a + 1 = 11$. Find the values of a , x , y , and k .

$$\begin{bmatrix} 2a + 1 & -6 \\ -7 & -3k \end{bmatrix} = \begin{bmatrix} 11 & x - 5 \\ 5 - 2y & 27 \end{bmatrix}$$



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Keep the steps in the summary below in mind as you solve equations that have variables on one side of the equation.



Key Concepts

Summary

Steps for Solving a Multi-Step Equation

- Step 1** Clear the equation of fractions and decimals.
Step 2 Use the Distributive Property to remove parentheses on each side.
Step 3 Combine like terms on each side.
Step 4 Undo addition or subtraction.
Step 5 Undo multiplication or division.

EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice*.

Practice and Problem Solving

A Practice by Example

Example 1
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Example 2
(page 127)

Example 3
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Example 4
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Example 5
(page 128)

Solve each equation. Check your answer.

- | | | |
|----------------------|------------------------|------------------------|
| 1. $4n - 2n = 18$ | 2. $y + y + 2 = 18$ | 3. $a + 6a - 9 = 30$ |
| 4. $5 - x - x = -1$ | 5. $72 + 4 - 14c = 36$ | 6. $13 = 5 - 13 + 3a$ |
| 7. $9 = -3 + n + 2n$ | 8. $7m - 3m - 6 = 6$ | 9. $-13 = 2b - b - 10$ |

Write an equation to model each situation. Solve your equation.

10. Two friends are renting an apartment. They pay the landlord the first month's rent. The landlord also requires them to pay an additional half of a month's rent for a security deposit. The total amount they pay the landlord before moving in is \$1725. What is the monthly rent?
11. You are fencing a rectangular puppy kennel with 25 ft of fence. The side of the kennel against your house does not need a fence. This side is 9 ft long. Find the dimensions of the kennel.

Solve each equation. Check your answer.

- | | | |
|---|--|---|
| 12. $2(8 + p) = 22$ | 13. $5(a - 1) = 35$ | 14. $15 = -3(2q - 1)$ |
| 15. $26 = 6(5 - a)$ | 16. $m + 5(m - 1) = 7$ | 17. $-4(x + 6) = -40$ |
| 18. $48 = 8(x + 2)$ | 19. $5(y - 3) = 19$ | 20. $5(2 + y) = 77$ |
| 21. $\frac{a}{7} - \frac{5}{7} = \frac{6}{7}$ | 22. $x - \frac{5}{8} = \frac{7}{8}$ | 23. $\frac{m}{6} - 7 = \frac{2}{3}$ |
| 24. $\frac{2}{3} + \frac{3k}{4} = \frac{71}{12}$ | 25. $4 + \frac{m}{8} = \frac{3}{4}$ | 26. $\frac{a}{2} + \frac{1}{5} = 17$ |
| 27. $\frac{1}{2} + \frac{7x}{10} = \frac{13}{20}$ | 28. $\frac{9y}{14} + \frac{3}{7} = \frac{9}{14}$ | 29. $\frac{1}{5} + \frac{3w}{15} = \frac{4}{5}$ |
| 30. $3m + 4.5m = 15$ | 31. $7.8y + 2 = 165.8$ | 32. $3.5 = 12s - 5s$ |
| 33. $1.06y - 3 = 0.71$ | 34. $0.11p + 1.5 = 2.49$ | 35. $25.24 = 5y + 3.89$ |
| 36. $1.12 + 1.25y = 8.62$ | 37. $1.025x + 2.458 = 7.583$ | 38. $0.25m + 0.1m = 9.8$ |

You can use a calculator to solve an equation. Using each side of the equation, you can graph two functions using the **Y=** screen. The x -value of the point of intersection is the solution of the equation.

GO for Help

For help entering functions in the Y= screen, see p. 125.

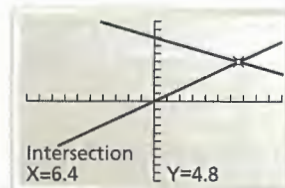
3 EXAMPLE Solving Using a Graphing Calculator

Solve $\frac{3}{4}m = 8 - \frac{1}{2}m$ using a graphing calculator.

Step 1 For Y_1 enter $\frac{3}{4}x$. For Y_2 enter $8 - \frac{1}{2}x$.

Step 2 Use the GRAPH feature to display the graph. You can adjust the window by using the ZOOM or WINDOW features.

Step 3 Use the CALC feature. Select intersect to find the point where the lines intersect.



- The lines intersect at (6.4, 4.8). The x -value 6.4 is the solution of the equation.

Quick Check

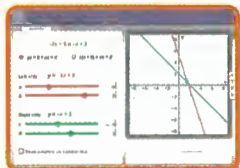
- 3 Solve $4 + \frac{1}{2}x = x - 1$ using a graphing calculator.

2

Special Cases: Identities and No Solutions

An equation has no solution if no value of the variable makes the equation true. The equation $2x = 2x + 1$ has no solution. An equation that is true for every value of the variable is an **identity**. The equation $2x = 2x$ is an identity.

Online active math



For: Solving Equations Activity
Use: Interactive Textbook, 3-3

4 EXAMPLE Identities and Equations with No Solutions

a. Solve $10 - 8a = 2(5 - 4a)$.

$$10 - 8a = 10 - 8a$$

$$10 - 8a + 8a = 10 - 8a + 8a$$

$$10 = 10 \text{ Always true!}$$

This equation is true for every value of a , so the equation is an identity.

b. Solve $6m - 5 = 7m + 7 - m$.

$$6m - 5 = 7m + 7 - m$$

$$6m - 5 = 6m + 7$$

$$6m - 5 - 6m = 6m + 7 - 6m$$

$$-5 = 7 \text{ Not true.}$$

This equation has no solution.

Quick Check

- 4 Determine whether each equation is an *identity* or whether it has *no solution*.

a. $9 + 5n = 5n - 1$

b. $9 + 5x = 7x + 9 - 2x$

EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice*.

Practice and Problem Solving

A Practice by Example

Example 1
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GO for Help

Solve each equation. Check your answer.

1. $6x - 2 = x + 13$

3. $4k - 3 = 3k + 4$

5. $8 - x = 2x - 1$

7. $3a + 4 = a + 18$

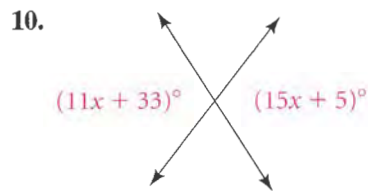
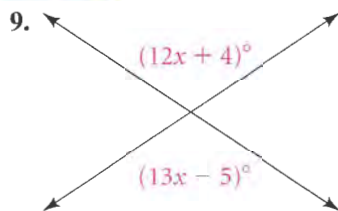
2. $5y - 3 = 2y + 12$

4. $5m + 3 = 3m + 9$

6. $2n - 5 = 8n + 7$

8. $6b + 14 = -7 - b$

Geometry Find the value of x .



Example 2
(page 135)

Write and solve an equation for each situation. Check your solution.

11. **Telephone Service** One telephone company charges \$16.95 per month and \$.05 per minute for local calls. Another company charges \$22.95 per month and \$.02 per minute for local calls. For what number of minutes of local calls per month is the cost of the plans the same?
12. **Fitness** One health club charges a \$44 sign-up fee and \$30 per month. Another health club charges a \$99 sign-up fee and \$25 per month. For what number of months is the cost the same?

Example 3
(page 136)

Solve each equation using a graphing calculator.

13. $7(3 - k) = -3k + 4$ 14. $a - 6 = 8 - (9 + a)$
15. $-\frac{1}{2}d + 2 = -4(d - \frac{1}{2})$ 16. $0.2n + 9 = 8(0.4n - 1)$

Example 4
(page 136)

17. a. Use the equation $9 - 6x = 3(3 - 2x)$. Substitute four different values for x and simplify.
b. What kind of equation is $9 - 6x = 3(3 - 2x)$?

Determine whether each equation is an identity or whether it has no solution.

18. $14 - (2q + 5) = -2q + 9$ 19. $6x + 1 = 6x - 8$
20. $-8x + 14 = -2(4x - 7)$ 21. $y - 5 = -(5 - y)$
22. $a - 4a = 2a + 1 - 5a$ 23. $9x + 3x - 10 = 3(3x + x)$

B Apply Your Skills



Solve each equation. If the equation is an identity, write *identity*. If it has no solution, write *no solution*.

24. $18x - 5 = 3(6x - 2)$ 25. $9 + 5a = 2a + 9$
26. $3(x - 4) = 3x - 12$ 27. $6x = 4(x + 5)$
28. $\frac{3}{5}k - \frac{1}{10}k = \frac{1}{2}k + 1$ 29. $0 = 0.98b + 0.02b - b$
30. $5m - 2(m + 2) = -(2m + 15)$ 31. $\frac{7}{8}w = \frac{4}{8}w + \frac{6}{8}w$

32. **Multiple Choice** A toy company spends \$1500 per day for factory expenses plus \$8 to make each teddy bear, like the one shown at the left. Which equation could be used to find the number of bears t the company has to sell in one day to equal its daily cost?

- (A) $1500 + 8t = 12$ (B) $12 + 8t = 1500$
(C) $1500 + 8t = 12t$ (D) $8t = 12t + 1500$

33. **Business** A company manufactures tote bags. The company spends \$1200 each day for overhead expenses plus \$9 per tote bag for labor and materials. The tote bags sell for \$25 each. How many tote bags must the company sell each day to equal its daily costs for overhead, labor, and materials? Write an equation and solve.

5 EXAMPLE Opposite-Direction Travel

Jane and Peter leave their home traveling in opposite directions on a straight road. Peter drives 15 mi/h faster than Jane. After 3 hours, they are 225 miles apart. Find Peter's rate and Jane's rate.

Define Let r = Jane's rate.
Then $r + 15$ = Peter's rate.

Relate

Person	Rate	Time	Distance
Jane	r	3	$3r$
Peter	$r + 15$	3	$3(r + 15)$

Jane's distance is $3r$. Peter's distance is $3(r + 15)$.

Write $3r + 3(r + 15) = 225$ The sum of Jane's and Peter's distances is the total distance, 225 miles.

$$3r + 3(r + 15) = 225$$

$$3r + 3r + 45 = 225$$

$$6r + 45 = 225$$

$$6r + 45 - 45 = 225 - 45$$

$$6r = 180$$

$$\frac{6r}{6} = \frac{180}{6}$$

$$r = 30$$

Use the Distributive Property.

Combine like terms.

Subtract 45 from each side.

Simplify.

Divide each side by 6.

Simplify.

- Jane's rate is 30 mi/h, and Peter's rate is 15 mi/h faster, which is 45 mi/h.



- 5** Sarah and John leave Perryville traveling in opposite directions on a straight road. Sarah drives 12 miles per hour faster than John. After 2 hours, they are 176 miles apart. Find Sarah's speed and John's speed.



Test-Taking Tip

When you grid an integer, right-align your answer so the place-value is clear.

		3	0
.	.	.	.
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice*.

Practice and Problem Solving

A Practice by Example

Example 1
(page 158)



- The length of a rectangle is 3 in. more than its width. The perimeter of the rectangle is 30 in.
 - Define a variable for the width.
 - Write an expression for the length in terms of the width.
 - Write an equation to find the width of the rectangle. Solve your equation.
 - What is the length of the rectangle?
- The length of a rectangle is 8 in. more than its width. The perimeter of the rectangle is 24 in. What are the width and length of the rectangle?
- The width of a rectangle is one half its length. The perimeter of the rectangle is 54 cm. What are the width and length of the rectangle?
- The length of a rectangular garden is 3 yd more than twice its width. The perimeter of the garden is 36 yd. What are the width and length of the garden?
- The sum of three consecutive integers is 915. What are the integers?

Example 2
(page 159)