

A radicand in the denominator of a radical expression may not be a perfect square. To simplify, you may need to **rationalize** the denominator. To do this, you multiply the numerator and the denominator by the same radical expression. You choose a radical expression that will make the denominator a perfect square.

7 EXAMPLE Rationalizing a Denominator

Simplify by rationalizing the denominator.

a. $\frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$ Multiply by $\frac{\sqrt{5}}{\sqrt{5}}$ to make the denominator a perfect square.
 $= \frac{2\sqrt{5}}{\sqrt{25}}$ Use the Multiplication Property of Square Roots.
 $= \frac{2\sqrt{5}}{5}$ Simplify $\sqrt{25}$.

b. $\frac{\sqrt{7}}{\sqrt{8n}} = \frac{\sqrt{7}}{\sqrt{8n}} \cdot \frac{\sqrt{2n}}{\sqrt{2n}}$ Multiply by $\frac{\sqrt{2n}}{\sqrt{2n}}$ to make the denominator a perfect square.
 $= \frac{\sqrt{14n}}{\sqrt{16n^2}}$ Use the Multiplication Property of Square Roots.
 $= \frac{\sqrt{14n}}{4n}$ Simplify $\sqrt{16n^2}$.



7 Simplify by rationalizing the denominator.

a. $\frac{3}{\sqrt{3}}$

b. $\frac{\sqrt{5}}{\sqrt{18t}}$

c. $\sqrt{\frac{7m}{10}}$

The summary below can help you determine whether a radical expression is in simplest radical form.

Key Concepts

Summary

Simplest Radical Form

A radical expression is in simplest radical form when all three statements are true.

- The radicand has no perfect-square factors other than 1.
- The radicand has no fractions.
- The denominator of a fraction has no radical.

EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice*.

Practice and Problem Solving

A Practice by Example

Examples 1, 2
(pages 616, 617)



Example 3
(page 617)

Simplify each radical expression.

1. $\sqrt{200}$

2. $\sqrt{98}$

3. $\sqrt{75}$

4. $-\sqrt{80}$

5. $-3\sqrt{120}$

6. $5\sqrt{320}$

7. $\sqrt{28n^2}$

8. $\sqrt{108b^4}$

9. $3\sqrt{12x^2}$

10. $\sqrt{4n^3}$

11. $\sqrt{20a^5}$

12. $-\sqrt{48b^4}$

13. $\sqrt{10} \cdot \sqrt{40}$

14. $3\sqrt{6} \cdot \sqrt{6}$

15. $\sqrt{22} \cdot \sqrt{11}$

16. $2\sqrt{18} \cdot 7\sqrt{6}$

17. $\sqrt{7} \cdot \sqrt{21}$

18. $-3\sqrt{20} \cdot \sqrt{15}$

19. $\sqrt{3n} \cdot \sqrt{24n}$

20. $2\sqrt{7t} \cdot \sqrt{14t}$

21. $\sqrt{3x} \cdot \sqrt{51x^3}$

22. $5\sqrt{8t} \cdot \sqrt{32t^5}$

23. $\sqrt{2a^2} \cdot \sqrt{9a^4}$

24. $-2\sqrt{6a^3} \cdot \sqrt{3a}$

Example 4
(page 617)

For Exercises 25–27, use the formula $d = \sqrt{1.5h}$ to approximate distance d in miles to a horizon when h is the height in feet of the viewer's eyes above the ground. Round your answer to the nearest mile.

25. Find the distance you can see to the horizon from a height of 6 feet.
 26. Find the distance you can see to the horizon from a height of 100 feet.
 27. Find the distance you can see to the horizon from a height of 200 feet.

Example 5
(page 618)

Simplify each radical expression.

- | | | | |
|--------------------------------|-------------------------------|--------------------------------|-------------------------------|
| 28. $\sqrt{\frac{21}{49}}$ | 29. $3\sqrt{\frac{3}{4}}$ | 30. $\sqrt{\frac{625}{100}}$ | 31. $\sqrt{\frac{120}{121}}$ |
| 32. $\sqrt{\frac{5}{9a^2}}$ | 33. $\sqrt{\frac{7}{16c^2}}$ | 34. $\sqrt{\frac{75a}{49}}$ | 35. $\sqrt{\frac{8n^3}{81}}$ |
| 36. $\sqrt{\frac{15}{5}}$ | 37. $\sqrt{\frac{54}{24}}$ | 38. $\sqrt{\frac{60}{5}}$ | 39. $-\sqrt{\frac{160}{8}}$ |
| 40. $\sqrt{\frac{140x^3}{5x}}$ | 41. $\sqrt{\frac{3s^3}{27s}}$ | 42. $\sqrt{\frac{30a^5}{40a}}$ | 43. $\sqrt{\frac{63y}{7y^3}}$ |

Example 6
(page 618)

Example 7
(page 619)

Simplify each radical expression by rationalizing the denominator.

- | | | | |
|--------------------------|----------------------------|-----------------------------------|-------------------------------------|
| 44. $\frac{3}{\sqrt{2}}$ | 45. $\frac{5}{\sqrt{5}}$ | 46. $\frac{\sqrt{3}}{\sqrt{7x}}$ | 47. $\frac{2\sqrt{2}}{\sqrt{5n}}$ |
| 48. $\frac{9}{\sqrt{8}}$ | 49. $\frac{12}{\sqrt{12}}$ | 50. $\frac{3\sqrt{2}}{\sqrt{9b}}$ | 51. $\frac{5\sqrt{11}}{\sqrt{20y}}$ |

B Apply Your Skills

Writing Explain why each radical expression is or is not in simplest radical form.

- | | | | |
|---------------------------|--------------------------|-----------------|------------------|
| 52. $\frac{13}{\sqrt{4}}$ | 53. $\frac{3}{\sqrt{3}}$ | 54. $4\sqrt{3}$ | 55. $5\sqrt{30}$ |
|---------------------------|--------------------------|-----------------|------------------|

56. Suppose a and b are positive integers.

a. Verify that if $a = 18$ and $b = 10$, then $\sqrt{a} \cdot \sqrt{b} = 6\sqrt{5}$.

b. **Open-Ended** Find two other pairs of positive integers a and b such that $\sqrt{a} \cdot \sqrt{b} = 6\sqrt{5}$.

Simplify each radical expression.

- | | | | |
|-----------------------------------|-------------------------------------|-----------------------------------|--------------------------------|
| 57. $\sqrt{12} \cdot \sqrt{75}$ | 58. $\sqrt{26 \cdot 2}$ | 59. $\frac{\sqrt{72}}{\sqrt{64}}$ | 60. $\frac{-2}{\sqrt{a^3}}$ |
| 61. $\frac{\sqrt{180}}{\sqrt{3}}$ | 62. $\frac{\sqrt{x^2}}{\sqrt{y^3}}$ | 63. $\frac{-3\sqrt{2}}{\sqrt{6}}$ | 64. $\sqrt{8} \cdot \sqrt{10}$ |
| 65. $\sqrt{20a^2b^3}$ | 66. $\sqrt{a^3b^5c^3}$ | 67. $\sqrt{\frac{3m}{16m^2}}$ | 68. $\frac{16a}{\sqrt{6a^3}}$ |

Solve each equation. Leave your answer in simplest radical form.

- | | | |
|------------------------|------------------------|-------------------------|
| 69. $x^2 + 6x - 9 = 0$ | 70. $n^2 - 2n + 1 = 5$ | 71. $3y^2 - 4y - 2 = 0$ |
|------------------------|------------------------|-------------------------|

72. a. Show work to verify that $\sqrt{50}$ equals $5\sqrt{2}$.



b. **Writing** Explain why $5\sqrt{2}$ is in simplest radical form.

73. **Open-Ended** What are three numbers whose square roots can be written in the form $a\sqrt{3}$ for some integer value of a ?

74. **Newspaper Layout** A square picture on the front page of a newspaper occupies an area of 24 in.^2 .

- a. Find the length of each side in simplest radical form.
 b. Calculate the length of each side to the nearest hundredth of an inch.



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C Challenge

Simplify each radical expression.

75. $\sqrt{24} \cdot \sqrt{2x} \cdot \sqrt{3x}$

76. $2b(\sqrt{5b})^2$

77. $\sqrt{45a^7} \cdot \sqrt{20a}$

78. **Physics** The time that a pendulum of a grandfather clock takes to swing back and forth one cycle is the period of the pendulum. The formula for finding the period T in seconds is $T = 2\pi\sqrt{\frac{L}{32}}$, where L is the length of the pendulum in feet. Find the period of a pendulum that is 8 feet long. Write your answer in terms of π .

**Test Prep****NY REGENTS****Multiple Choice**79. Simplify $\sqrt{80}$.

A. $10\sqrt{8}$

B. $8\sqrt{10}$

C. $4\sqrt{5}$

D. 40

80. Simplify $5\sqrt{3x^2} \cdot \sqrt{6x}$.

F. $15x\sqrt{2x}$

G. $5x\sqrt{18x}$

H. $3x\sqrt{10x}$

J. $6x\sqrt{5x}$

81. Which of the following equals $\frac{2}{3}$?

A. $\sqrt{\frac{9}{25}}$

B. $\sqrt{\frac{20}{45}}$

C. $2\sqrt{\frac{4}{27}}$

D. $\sqrt{\frac{6}{9}}$

82. Which of the following equals $1.5\sqrt{0.038}$?

F. $150\sqrt{3.8}$

G. $15\sqrt{3.8}$

H. $15\sqrt{0.38}$

J. $15\sqrt{0.00038}$

Short Response83. A square window has an area of 96 ft^2 . What is the length of each side of the window in simplest radical form? Show your work.**Mixed Review****Lesson 10-8**

Which kind of function best models the data in each table? Write an equation to model the data.

84.

x	y
-1	0.2
0	0
1	0.2
2	0.8
3	1.8
4	3.2

85.

x	y
-1	1.6
0	4
1	10
2	25
3	62.5
4	156.25

86.

x	y
-1	11.2
0	7
1	2.8
2	-1.4
3	-5.6
4	-9.8

Lesson 10-2

Graph each function. Label the axis of symmetry and the vertex.

87. $f(x) = x^2 + 8x - 4$

88. $y = x^2 - 10x + 7$

89. $y = 3x^2 + 12x - 5$

Lesson 9-1

Simplify. Write each answer in standard form.

90. $(n^2 + 5n - 1) + (2n^2 + 6)$

91. $(4v^2 + 8v - 2) - (v^2 + 9v + 7)$

92. $(5t^3 - 14t) + (8t^2 - 11)$

93. $(2b^2 - 12b - 8) - (5b^2 + 11b + 13)$