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Factoring a Monomial From a Polynomial

Vocabulary Tip

The greatest common factor (GCF) is the greatest factor that divides evenly into each term of an expression. See Skills Handbook page 755.

Factoring a polynomial reverses the multiplication process. To factor a monomial from a polynomial, first find the greatest common factor (GCF) of its terms.

2 EXAMPLE Finding the Greatest Common Factor

Find the GCF of the terms of $4x^3 + 12x^2 - 8x$.

List the prime factors of each term. Identify the factors common to all terms.

$$4x^3 = 2 \cdot 2 \cdot x \cdot x \cdot x$$

$$12x^2 = 2 \cdot 2 \cdot 3 \cdot x \cdot x$$

$$8x = 2 \cdot 2 \cdot 2 \cdot x$$

- The GCF is $2 \cdot 2 \cdot x$ or $4x$.



- 2 Find the GCF of the terms of each polynomial.

a. $5v^5 + 10v^3$

b. $3t^2 - 18$

c. $4b^3 - 2b^2 - 6b$

To factor a polynomial completely, you must factor until there are no common factors other than 1.

3 EXAMPLE Factoring Out a Monomial

Factor $3x^3 - 12x^2 + 15x$.

Step 1 Find the GCF.

$$3x^3 = 3 \cdot x \cdot x \cdot x$$

$$12x^2 = 2 \cdot 2 \cdot 3 \cdot x \cdot x$$

$$15x = 3 \cdot 5 \cdot x$$

- The GCF is $3 \cdot x$ or $3x$.

Step 2 Factor out the GCF.

$$3x^3 - 12x^2 + 15x$$

$$= 3x(x^2) + 3x(-4x) + 3x(5)$$

$$= 3x(x^2 - 4x + 5)$$



- 3 Use the GCF to factor each polynomial.

a. $8x^2 - 12x$

b. $5d^3 + 10d$

c. $6m^3 - 12m^2 - 24m$

EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice*.

Practice and Problem Solving

A Practice by Example

Example 1
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Simplify each product.

1. $8m(m + 6)$

2. $(x + 10)3x$

3. $9k(7k + 4)$

4. $-5a(a - 1)$

5. $2x^2(9 + x)$

6. $-p^2(p - 11)$

7. $2x(6x^3 - x^2 + 5x)$

8. $4y^2(9y^3 + 8y^2 - 11)$

9. $-5c^3(9c^2 - 8c - 5)$

10. $-7q^2(6q^5 - 2q - 7)$

11. $-3g^7(g^4 - 6g^2 + 5)$

12. $-4x^6(10x^3 + 3x^2 - 7)$

Example 2
(page 501)

Find the GCF of the terms of each polynomial.

13. $15w + 21$

14. $6a^2 - 8a$

15. $36v + 24$

16. $x^3 + 7x^2 - 5x$

17. $5b^3 + 15b - 30$

18. $9x^3 - 6x^2 + 12x$

Example 3
(page 501)

Factor each polynomial.

19. $6x - 4$

20. $v^2 + 4v$

21. $10x^3 - 25x^2 + 20$

22. $2t^2 - 10t^4$

23. $15n^3 - 3n^2 + 12n$

24. $6p^6 + 24p^5 + 18p^3$

B Apply Your Skills

25. **Error Analysis** Kevin said that $-2x(4x - 3) = -8x^2 - 6x$. Karla said that $-2x(4x - 3) = -8x^2 + 6x$. Who is correct? Explain.

26. **Open-Ended** Write a polynomial that has a common factor in each term. Factor your polynomial.

Simplify. Write in standard form.

27. $-3a(4a^2 - 5a + 9)$

28. $-7p^2(-2p^3 + 5p)$

29. $12c(-5c^2 + 3c - 4)$

30. $y(y + 3) - 5y(y - 2)$

31. $x^2(x + 1) - x(x^2 - 1)$

32. $4t(3t^2 - 4t) - t(7t)$



33. **Building Models** Suppose you are building a model of the square castle shown at the left. The moat of the model castle is made of blue paper.
a. Find the area of the moat using the diagram with the photo.
b. Write your answer in factored form.

Factor each polynomial.

34. $9m^{12} - 36m^7 + 81m^5$

35. $24x^3 - 96x^2 + 48x$

36. $16n^3 + 48n^2 - 80n$

37. $5x^4 + 4x^3 + 3x^2$

38. $13ab^3 + 39a^2b^4$

39. $7g^2k^3 - 35g^5k^2$

40. **Critical Thinking** The GCF of two numbers p and q is 5. What is the GCF of p^2 and q^2 ? Explain your answer.

41. a. Factor $n^2 - n$.
b. **Writing** Suppose n is an integer. Is $n^2 - n$ always, sometimes, or never even? Justify your answer.

42. A triangular number is a number you can represent with a triangular arrangement of objects. A triangular number can also be written as a product of two factors, as in the table.

a. Find the values of a , b , c , and d , and then write an expression in factored form for the n th triangular number.

	1	2	3	4
Triangular Number	1	3	6	10
Factored Form	$\frac{a}{2}(a + 1)$	$\frac{b}{2}(b + 1)$	$\frac{c}{2}(c + 1)$	$\frac{d}{2}(d + 1)$

b. Use the expression you wrote to find the 100th triangular number.

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C Challenge

43. a. **Geometry** How many sides does the polygon have? How many of its diagonals come from one vertex?
b. Suppose a polygon has n sides. How many diagonals will it have from one vertex?
c. The number of diagonals from all the vertices is $\frac{n}{2}(n - 3)$. Multiply the two factors.
d. For a polygon with 8 sides, what is the total number of diagonals that can be drawn from the vertices?

