

FOIL works when you multiply two binomials, but it is not helpful when multiplying a trinomial and a binomial. You can use the vertical method or the horizontal method to distribute each term in such factors.

## 4 EXAMPLE Multiplying a Trinomial and a Binomial

Simplify the product  $(4x^2 + x - 6)(2x - 3)$ .

**Method 1** Multiply using the vertical method.

$$\begin{array}{r}
 4x^2 + x - 6 \\
 \underline{\phantom{4x^2 + x - 6} 2x - 3} \\
 -12x^2 - 3x + 18 \quad \text{Multiply by } -3. \\
 8x^3 + 2x^2 - 12x \quad \text{Multiply by } 2x. \\
 \hline
 8x^3 - 10x^2 - 15x + 18 \quad \text{Add like terms.}
 \end{array}$$

**Method 2** Multiply using the horizontal method.

$$(2x - 3)(4x^2 + x - 6)$$

$$\begin{aligned}
 &= 2x(4x^2) + 2x(x) + 2x(-6) - 3(4x^2) - 3(x) - 3(-6) \\
 &= 8x^3 + 2x^2 - 12x - 12x^2 - 3x + 18 \\
 &= 8x^3 - 10x^2 - 15x + 18 \quad \text{Add like terms.}
 \end{aligned}$$

- The product is  $8x^3 - 10x^2 - 15x + 18$ .



- 4 Simplify  $(6n - 8)(2n^2 + n + 7)$  using both methods shown in Example 4.

## EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice*.

### Practice and Problem Solving

#### A Practice by Example

**Example 1**  
(page 505)



Copy and fill in each blank.

- $(5a + 2)(6a - 1) = \blacksquare a^2 + 7a - 2$
- $(3c - 7)(2c - 5) = 6c^2 - 29c + \blacksquare$
- $(z - 4)(2z + 1) = 2z^2 - \blacksquare z - 4$
- $(2x + 9)(x + 2) = 2x^2 + \blacksquare x + 18$

Simplify each product using the Distributive Property.

- $(x + 2)(x + 5)$
- $(h + 3)(h + 4)$
- $(k + 7)(k - 6)$
- $(a - 8)(a - 9)$
- $(2x - 1)(x + 2)$
- $(2y + 5)(y - 3)$

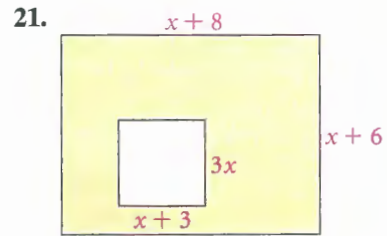
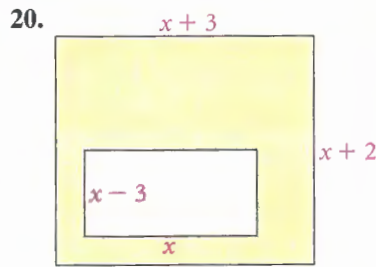
**Example 2**  
(page 506)

Simplify each product using FOIL.

- $(r + 6)(r - 4)$
- $(y + 4)(5y - 8)$
- $(x + 6)(x - 7)$
- $(m - 6)(m - 9)$
- $(4b - 2)(b + 3)$
- $(8w + 2)(w + 5)$
- $(x - 7)(x + 9)$
- $(a + 11)(a + 5)$
- $(p - 1)(p + 10)$

**Example 3**  
(page 506)

**Geometry** Find an expression for the area of each shaded region. Simplify.



**Example 4**  
(page 507)

**Simplify. Use the vertical method.**

22.  $(x + 9)(x^2 - 4x + 1)$

23.  $(a - 4)(a^2 - 2a + 1)$

24.  $(g - 3)(2g^2 + 3g + 3)$

25.  $(k + 8)(3k^2 - 5k + 7)$

**Simplify. Use the horizontal method.**

26.  $(x^2 + 2x + 1)(9x - 3)$

27.  $(t^2 - 6t + 3)(2t - 5)$

28.  $(7p^2 + 5p - 1)(8p + 9)$

29.  $(12w^2 - w - 1)(4w - 2)$

**B Apply Your Skills**

**Simplify each product. Write in standard form.**

30.  $(p - 7)(p + 8)$

31.  $(-7 + p)(8 + p)$

32.  $(p^2 - 7)(p + 8)$

33.  $(5c - 9)(5c + 1)$

34.  $(n^2 + 3)(n + 11)$

35.  $(3k^2 + 2)(k + 5k^2)$

36.  $(6h - 1)(4h^2 + h + 3)$

37.  $(9y^2 + 2)(y^2 - y - 1)$

38.  $(8q - 4)(6q^2 + q + 1)$

**GO Online**  
**Homework Video Tutor**

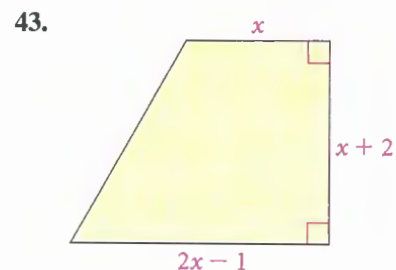
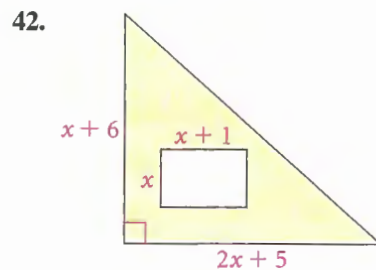
Visit: PHSchool.com  
Web Code: ate-0903

39. **Construction** You are planning a rectangular garden. Its length is twice its width  $x$ . You want a walkway 2 ft wide around the garden.
- Write an expression for the area of the garden and walk.
  - Write an expression for the area of the walk only.
  - You have enough gravel to cover  $76 \text{ ft}^2$  and want to use it all on the walk. How big should you make the garden?

40. **Open-Ended** Write a binomial and a trinomial. Find their product.

41. **Writing** Which method do you prefer for multiplying a binomial and a trinomial? Explain.

**Geometry** Write an expression for the area of each shaded region. Write your answer in simplest form.



44. a. Simplify each pair of products.

i.  $(x + 1)(x + 1)$   
 $11 \cdot 11$

ii.  $(x + 1)(x + 2)$   
 $11 \cdot 12$

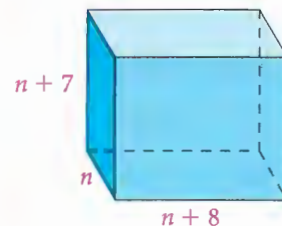
iii.  $(x + 1)(x + 3)$   
 $11 \cdot 13$

- b. **Critical Thinking** What are the similarities between the two answers in each pair of products?

**GO for Help**

For a guide to solving Exercise 42, see p. 511.

45. **Geometry** Use the formula  $V = \ell wh$  to write a polynomial in standard form for the volume of the box.



46. **Multiple Choice** If  $n$  represents an even number, which expression represents the product of the next two even numbers?

- (A)  $n^2 + 3n + 2$  (B)  $2n + 3$  (C)  $2n + 6$  (D)  $n^2 + 6n + 8$

**C Challenge**

For Exercises 47–49, each expression represents the side length of a cube. Write an expression in standard form for the surface area of each cube.

47.  $x + 3$                       48.  $4t + 1$                       49.  $2w^2 + 7$



50. a. **Vegetable Consumption** Multiply the expressions on the right side of each equation to create a model for the total number of pounds of fresh vegetables  $V(t)$  consumed in a year in the United States.

$C(t) = 3.2t + 157$       the U.S. annual per capita consumption of fresh vegetables, in pounds, from 1990 to 1997

$P(t) = 3.3t + 250$       the U.S. population, in millions, from 1990 to 1997

- b. Evaluate the equation you found in part (a) with  $t = 5$  to find the total vegetable consumption for 1995. ( $t = 0$  corresponds to the year 1990.)

51. **Financial Planning** Suppose you deposit \$2000 for college in a savings account that has an annual interest rate  $r$ . At the end of three years, the value of your account will be  $2000(1 + r)^3$  dollars.

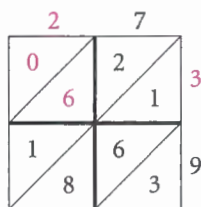
- a. Rewrite the expression  $2000(1 + r)^3$  by finding the product  $2000(1 + r)(1 + r)(1 + r)$ . Write your answer in standard form.  
b. Find the amount of money in the account if the interest rate is 3%.

**Real-World Connection**

In 2003, the U.S. consumption of fresh tomatoes was 15.4 lb per person.

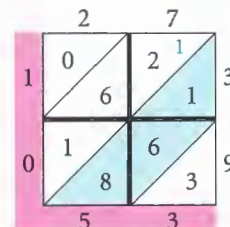
For Exercises 52–55, find each product using lattice multiplication, which is explained below.

Lattice multiplication probably originated in India in the twelfth century. It came into use in Italy in the fourteenth century.



This example shows  $27 \cdot 39$ . Each number is treated as a binomial. The four products are placed in the small, diagonally split squares. The product of 2 and 3, shown in red, is 6. The first square shows 0/6, which indicates 6. The product of 7 and 3 is 21. The second square shows 2/1.

The products are totaled diagonally. For the diagonal shaded blue, the tens place of the sum  $1 + 6 + 8$  is carried into the diagonal above and added into that diagonal:  $1 + (2 + 6 + 1)$ . The product 1053 appears down the left side of the lattice and across the bottom.



52.  $14 \cdot 72$                       53.  $53 \cdot 87$   
54.  $91 \cdot 64$                       55.  $38 \cdot 64$



Multiple Choice

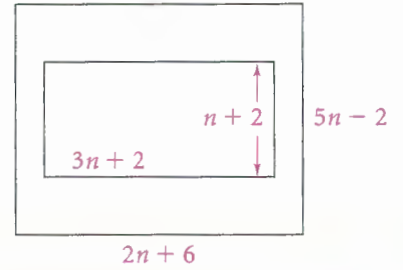
56.  $(n - 1)(n - 4)$  is equivalent to which expression?  
 A.  $n^2 - 5n + 4$                                   B.  $n^2 - 3n + 4$   
 C.  $n^2 + 3n + 4$                                   D.  $n^2 - 5n - 5$
57.  $(8k - 3)(k^2 - k + 1)$  is equivalent to which expression?  
 F.  $8k^3 + 11k^2 - 11k - 3$                                   G.  $9k^3 - 8k^2 + 8k - 2$   
 H.  $8k^3 - 11k^2 + 11k - 3$                                   J.  $9k^3 - 3k^2 + 3k - 3$
58. Which of the following products is always odd for integer values of  $n$ ?  
 A.  $(n + 1)(n + 1)$                                   B.  $(2n - 1)(2n + 1)$   
 C.  $(2n - 1)(n + 1)$                                   D.  $(2n + 1)(n - 1)$

Short Response

59. Explain how to find the product of  $(4v - 1)(2v^2 + v + 1)$ , and simplify.

Extended Response

60. Find an expression for the area of the shaded region. Show your work.



Mixed Review



Lesson 9-2

Simplify each product.

61.  $4v(5v - 7)$                                   62.  $(c - 9)3c$                                   63.  $8t^2(t + 6)$                                   64.  $y(3y - 10)$   
 65.  $5x^2(11 - x)$                                   66.  $-t^3(6t - 1)$                                   67.  $4r(3 - r^5)$                                   68.  $9b^2(b^3 + 2b)$

Factor.

69.  $5w + 45$                                   70.  $3x^2 - 11x$                                   71.  $4a^2 + 12a$                                   72.  $9n^2 - n^3$   
 73.  $34t - 51$                                   74.  $63v^2 + 45v$                                   75.  $25m - 60m^3$                                   76.  $11k + 77k^6$

Lesson 8-5

Simplify each expression.

77.  $\frac{3^5}{3^2}$                                   78.  $\frac{3^2}{3^5}$                                   79.  $\frac{y^{12}}{y^8}$                                   80.  $\frac{2w^{-3}}{6w^2}$                                   81.  $\frac{x^{-8}}{2x^3}$   
 82.  $(\frac{5}{3})^{-1}$                                   83.  $(\frac{5}{3})^{-2}$                                   84.  $(\frac{5}{3})^0$                                   85.  $(\frac{4x}{7})^{-2}$                                   86.  $(\frac{y^{-2}}{8})^{-2}$



Checkpoint Quiz 1

Lessons 9-1 through 9-3

Simplify each expression.

1.  $(4x^2 + x + 3) + (5x^2 + 9x - 2)$                                   2.  $(7b^2 - 5b + 3) - (b^2 + 8b - 6)$   
 3.  $3w(12w - 1) - 8w$                                   4.  $6k(4k + k^2) + 9k(2k - 6k^2)$   
 5.  $(x + 3)(x - 5)$                                   6.  $(2n^3 - 5)(6n^2 + n)$                                   7.  $(g^2 + 4)(4g^2 + 8g - 9)$

Factor each polynomial.

8.  $12y^2 - 10$                                   9.  $5t^6 + 25t^3 - 10t$                                   10.  $18v^4 + 27v^3 + 36v^2$

You can use the difference of squares to calculate products using mental math.

### 5 EXAMPLE Mental Math

Find  $82 \cdot 78$ .

$$\begin{aligned} 82 \cdot 78 &= (80 + 2)(80 - 2) && \text{Express each factor using 80 and 2.} \\ &= 80^2 - 2^2 && \text{Find the difference of squares.} \\ &= 6400 - 4 = 6396 && \text{Simplify.} \end{aligned}$$



5 Find each product.

- a.  $18 \cdot 22$       b.  $19 \cdot 21$       c.  $59 \cdot 61$       d.  $87 \cdot 93$

## EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice*.

### Practice and Problem Solving

#### A Practice by Example

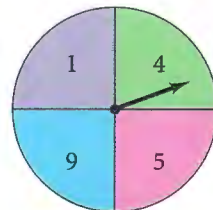
Examples 1, 2  
(page 513)



Find each square.

1.  $(c + 1)^2$       2.  $(x + 4)^2$       3.  $(2v + 11)^2$       4.  $(3m + 7)^2$   
5.  $(w - 12)^2$       6.  $(b - 5)^2$       7.  $(6x - 8)^2$       8.  $(9j - 2)^2$

9. **Games** Suppose you play a game with two spinners like the one shown at the right. Let  $C$  represent spinning an even number. Let  $D$  represent spinning an odd number. The probability of  $C$  is  $\frac{1}{4}$ . The probability of  $D$  is  $\frac{3}{4}$ .



- a. Simplify  $(\frac{1}{4}C + \frac{3}{4}D)^2$ .  
b. Find  $P(C \text{ and } C)$ .  
c. How does the answer in part (b) relate to the polynomial in part (a)?

Example 3  
(page 514)

**Mental Math** Find each square.

10.  $61^2$       11.  $99^2$       12.  $48^2$       13.  $302^2$       14.  $499^2$

Example 4  
(page 514)

Find each product.

15.  $(x + 4)(x - 4)$       16.  $(a + 8)(a - 8)$       17.  $(d + 7)(d - 7)$   
18.  $(h + 15)(h - 15)$       19.  $(y + 12)(y - 12)$       20.  $(k + 5)(k - 5)$

Example 5  
(page 515)

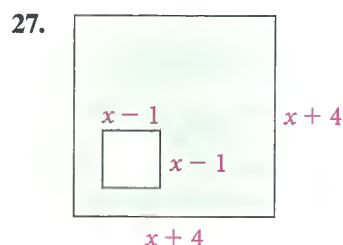
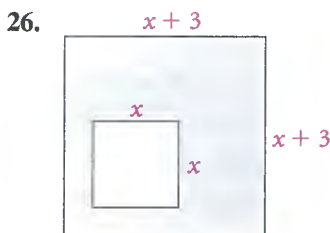
**Mental Math** Find each product.

21.  $31 \cdot 29$       22.  $89 \cdot 91$       23.  $52 \cdot 48$       24.  $197 \cdot 203$       25.  $299 \cdot 301$

#### B Apply Your Skills



**Geometry** Find an expression for the area of each shaded region. Write your answers in standard form.



Find each square.

28.  $(x + 3y)^2$       29.  $(5p - q)^2$       30.  $(6m + n)^2$       31.  $(x - 7y)^2$   
 32.  $(4k + 7j)^2$       33.  $(2y - 9x)^2$       34.  $(3w + 10t)^2$       35.  $(6a + 11b)^2$   
 36.  $(5p - 6q)^2$       37.  $(6h - 8p)^2$       38.  $(y^5 - 9x^4)^2$       39.  $(8k + 4h)^2$



**Real-World Connection**

The cow in the photo shows a typical roan coat.



40. **Biology** The coat color of shorthorn cattle is determined by two genes, Red  $R$  and White  $W$ .  $RR$  produces red,  $WW$  produces white, and  $RW$  produces a third type of coat color called roan.

	$R$	$W$
$R$	$RR$	$RW$
$W$	$RW$	$WW$

- a. Model the Punnett square with the square of a binomial.  
 b. If both parents have  $RW$ , what is the probability the offspring will also be  $RW$ ?  
 c. Write an expression to model a situation where one parent is  $RW$  while the other is  $RR$ .  
 d. What is the probability that the offspring of the parents in step (c) will have a white coat?

41. a. Copy and complete the table.  
 b. Describe any patterns you see.  
 c. **Writing** How does the difference of squares account for the pattern in the table?

$4^2 = 16$	$3 \cdot 5 = 15$
$5^2 = \blacksquare$	$4 \cdot 6 = 24$
$6^2 = \blacksquare$	$5 \cdot 7 = \blacksquare$
$7^2 = \blacksquare$	$6 \cdot 8 = \blacksquare$

42. **Open-Ended** Give a counterexample to show that  $(x + y)^2 = x^2 + y^2$  is false.  
 43. **Critical Thinking** Does  $(3\frac{1}{2})^2 = 9\frac{1}{4}$ ? Explain.

Find each product.

44.  $(3y + 5w)(3y - 5w)$       45.  $(p + 9q)(p - 9q)$       46.  $(2d + 7g)(2d - 7g)$   
 47.  $(7b - 8c)(7b + 8c)$       48.  $(g + 7h)(g - 7h)$       49.  $(g^3 + 7h^2)(g^3 - 7h^2)$   
 50.  $(2a^2 + b)(2a^2 - b)$       51.  $(11x - y^3)(11x + y^3)$       52.  $(4k - 3h^2)(4k + 3h^2)$

**Challenge**

53. Write the expression  $(a + b + c)^2$  in standard form.  
 54. **Games** Suppose you play a game by tossing 3 coins. You can find the probabilities by simplifying  $(\frac{1}{2}H + \frac{1}{2}T)^3$ .  
 a. Simplify the expression.  
 b. Use the answer you found in part (a) to find the probability of getting a head and two tails ( $HT^2$ ).

55. **Number Theory** You can use factoring to show that the sum of two multiples of 3 is also a multiple of 3.

If  $m$  and  $n$  are integers, then  $3n$  and  $3m$  are multiples of three.

$$3m + 3n = 3(m + n)$$

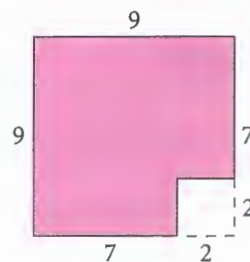
Since  $(m + n)$  is an integer,  $3(m + n)$  is a multiple of three.

- a. Show that if a number is one more than a multiple of 3, then its square is also one more than a multiple of 3.  
 b. **Reasoning** If a number is two more than a multiple of 3, is its square also two more than a multiple of 3? Explain.  
 56. The formula  $V = \frac{4}{3}\pi r^3$  gives the volume of a sphere. Find the formula for the volume of a sphere that has a radius 3 more than  $r$ . Write your answer in standard form.

**GO Online**  
**Homework Video Tutor**  
 Visit: PHSchool.com  
 Web Code: ate-0904

57. The area of the shaded region in the diagram is  $9^2 - 2^2$ .

- a. Copy the figure. Make a single cut across the shaded region and reassemble it to show that  $9^2 - 2^2 = (9 - 2)(9 + 2)$ .
- b. Draw your reassembled figure. Include its dimensions.



Test Prep

Multiple Choice

58. Which value of  $a$  makes  $(9x - 1)^2 = ax^2 - 18x + 1$  true?  
 A. 9                      B. 18                      C. 64                      D. 81
59. Which value of  $n$  makes  $(b^7 + 2)^2 = b^n + 4b^7 + 4$  true?  
 F. 14                      G. 28                      H. 42                      J. 49
60. Simplify  $(x - 1)^2 + (x + 1)^2$ .  
 A.  $2x$                       B.  $-2x$                       C.  $2x^2 + 2$                       D.  $2x^2$
61. Find the product of  $(2x - 3)$ ,  $(4x^2 + 9)$ , and  $(2x + 3)$ .  
 F.  $16x^2 + 18$                       G.  $16x^4 + 18$   
 H.  $16x^4 - 81$                       J.  $64x^4 - 81$
62. Which of the following correctly shows how to use the Difference of Squares rule to multiply 17 and 23?  
 A.  $17 \cdot 23 = (16 + 1)(16 + 7)$   
      $= 16^2 + (1 \cdot 7)$   
      $= 256 + 7$   
      $= 263$   
 B.  $17 \cdot 23 = (20 - 3)(20 + 3)$   
      $= 20^2 - 9$   
      $= 400 - 9$   
      $= 391$   
 C.  $17 \cdot 23 = (19 - 2)(19 + 4)$   
      $= 19^2 - (2 \cdot 4)$   
      $= 361 - 8$   
      $= 353$   
 D.  $17 \cdot 23 = (18 - 1)(22 + 1)$   
      $= 18 \cdot 22 - 1^2$   
      $= 396 - 1$   
      $= 395$

Short Response

63. Explain how to compute the  $xy$  term of the product  $(3x - 4y)^2$ .

Mixed Review



Lesson 9-3

Find each product.

64.  $(k + 7)(k - 9)$                       65.  $(2x - 11)(x - 6)$                       66.  $(5p + 4)(3p - 1)$   
 67.  $(3y + 1)(y + 1)$                       68.  $(4h - 2)(6h + 1)$                       69.  $(9b + 7)(8b + 2)$   
 70.  $(2w^2 + 5)(w + 8)$                       71.  $(r - 7)(r^2 + 3r - 9)$                       72.  $(5m^2 - 2)(6m^3 + 4m)$

Lesson 8-2

Write each number in scientific notation.

73. 8713                      74. 0.031                      75. 68,952                      76. 1.2 million  
 77. 11                      78. 523                      79. 6 billion                      80. 0.72